

ESSENTIAL FEM STATEMENTS APPLIED TO STRUCTURAL MECHANICS PROBLEMS. Part 1

In the article, the author shares his classification of FEM statements that may serve as a guide in respect of the huge number of works that are published and being published with a view to the FEM efficiency improvement. The author provides a summarized history of the finite element method, and classifies its configurations and versions. The author also provides FEM statements applicable to the deflection method. Derivation of the rigidity matrix designated for shaft-based finite elements is demonstrated in the article. The author employs one-dimensional framing as an example aimed to demonstrate the convergence of the FEM method in terms of deflections, if the finite element grid is refined. However it is also noteworthy that in the event of a fine grid, the finite element designed for plates does not coincide with the finite element of a thin plate designed as the initial physical model. However, the system of equations, provided by the author, takes account of the influence produced by the load onto the finite element and generates the exact solution irrespective of any finite values of the length that are equal to the physical model of a finite element.

Key words: FEM, classification, statement, deflection method, structural mechanics, force method.

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